Differential Equations
Summer 2014

Things to Review

Integration

1) $\int x e^x \, dx$ Integration by parts

$= \boxed{x e^x - e^x + c}$

$$\int u \, dv = uv - \int v \, du$$

$u = x$ $dv = e^x$

$du = dx$ $v = e^x$

2) $\int \ln x \, dx$ Integration by Parts too.

note $\int \ln x \, dx \neq \frac{1}{x} + c$

$u = \ln x$ $dv = dx$

$du = \frac{1}{x} dx$ $v = x$

$= x \ln x - \int x \cdot \frac{1}{x} dx = \boxed{x \ln x - x + c}$

not trig sub

3) $\displaystyle\int \frac{1}{x^2-1}\,dx$ $\boxed{\text{partial fractions}}$

$$\int \left(\frac{A}{x+1} + \frac{B}{x-1}\right) dx$$ where A & B are real constants

$$= A\ln|x+1| + B\ln|x-1| + C$$

then find $\underline{A}$ & $\underline{B}$

$$\frac{A}{x+1} + \frac{B}{x-1} = \frac{1}{x^2-1}$$

partial fraction decomposition

$$A(x-1) + B(x+1) = 1$$
$$Ax - A + Bx + B = 1$$
$$x(A+B) + (B-A) = 1$$

$A+B=0$ $\quad$ $B-A=1$

$A=-B$ $\quad$ $B-(-B)=1$

$2B=1$

$\boxed{B=\tfrac{1}{2} \quad A=-\tfrac{1}{2}}$

so $\boxed{-\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C}$

<u>practice:</u> $\displaystyle\int \frac{x+2}{(x-1)(x^2+1)}\,dx$

3 partial fractions needed.

Review of Solving Polynomial Equations

$\boxed{m^2 - 1 = 0}$ or $m^2 - 1 = 0$

$(m+1)(m-1) = 0$ $\qquad$ $m^2 = 1$

$m = \pm 1$ $\qquad$ $m = \pm\sqrt{1} = \pm 1$

$\boxed{\text{Solve } m^3 - 2m^2 + m = 0}$

$m(m^2 - 2m + 1) = 0$ $\qquad$ $m = 0, \pm 1$

$m(m-1)(m-1) = 0$

$\boxed{\text{Solve } m^3 - 2m^2 + 1 = 0}$??

recall $(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

$(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$

(not for this one)

How do we factor this? $\longrightarrow$ rational zeros!
(one way)

or graphing calculator
(look for x intercepts)

rational zero check

$\boxed{m=1}$ $\quad 1^3 - 2(1)^2 + 1 = 0$

$0 = 0$ $\qquad$ so $m = 1$ is a zero

$m = -1$ $\quad (-1)^3 - 2(-1)^2 + 1 = 0$ $\qquad$ $m - 1$ is a factor

$-2 \neq 0$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & 0 & 1 \\ & & 1 & -1 & -1 \\ \hline & 1 & -1 & -1 & 0 \end{array}$$

so $(m-1)\underbrace{(m^2 - m - 1)}_{\text{use quad. formula on this part}} = 0$

since $m^3 - 2m^2 + 1 = (m-1)(\ \ ?\ \ )$

$a=1 \quad b=-1 \quad c=-1$

$m^2 - m - 1 = 0$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$m = \frac{+1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$$

$$\boxed{m} = \frac{1 \pm \sqrt{1+4}}{2} = \boxed{\frac{1 \pm \sqrt{5}}{2}}$$

Recall discriminant $D = b^2 - 4ac$
$= (1) - 4(1)(-1) = 5$

Since $D > 0 \Rightarrow$ 2 real solutions !

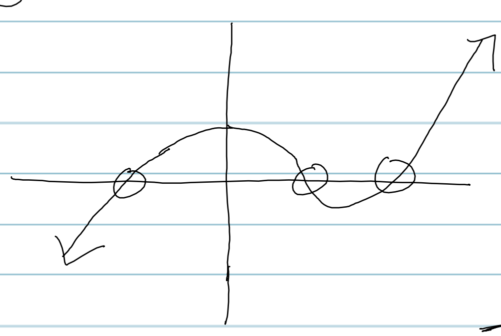
Final Answer $\quad \boxed{m = 1, \; m = \frac{1+\sqrt{5}}{2}, \; m = \frac{1-\sqrt{5}}{2}}$

corresponds to an auxillary equation
(in chapter 4)

using the graphing calculator

$y = x^3 - 2x^2 + 1$

$X_{min} = -5$
$X_{max} = 5$
$y_{min} = -5$
$y_{max} = 5$



use intercept func.

$-.62$ & 1.6 + 1

Now onto Differential Equations

why/where → circuits, electrical engineering

What does one look like?

~~A population~~ The rate of change of a population with respect to time varies directly with the population itself.

Leibniz notation

$\frac{dP}{dt}$ = rate of change P = population size, t = time

$$\frac{dP}{dt} = k \cdot P$$ where k is a real constant

~~wh~~ we have a function & its derivative.

$\frac{dP}{dt} = k \cdot P$ $P' = k \cdot P$ or $\dot{P} = k \cdot P$ (from physics)

now Solve it

treat the rate of change as if it were a fraction (which it is not)

$$\frac{dP}{dt} = kP$$

$$dP = kP\,dt$$

$$\frac{dP}{P} = k \cdot dt$$

$$\int \frac{1}{P}\,dP = \int k\,dt$$

$$\ln|P| = kt + C$$

combo of left & right integral

this is like:

$$\ln|p| + c_1 = kt + c_2$$

$$\ln|p| = kt + \underbrace{c_1 + c_2}_{c}$$

$$\boxed{\ln|p| = kt + c}$$

This doesn't tell us exactly what p is. It is an implicit form.

$$e^{\ln|p|} = e^{kt+c}$$

$$\boxed{p = e^{kt+c}} = e^{kt}\underbrace{e^{c}}_{c} = \boxed{Ce^{kt}}$$

another constant

$$\Rightarrow \quad P(t) = Ce^{kt}$$

frequently C is called A_0

$$\Rightarrow \quad \boxed{P(t) = A_0 e^{kt}}$$

(you may call it any constant value)

This is the college algebra exponential model!

goal → 1) combine constants when possible
2) make the constants as clean & simple as possible (efficient)